

The phase diagram of QCD, third families of proto-compact stars, and the possibility of core-collapse supernova explosions

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(Dated: November 23, 2015)

A phase transition (PT) to quark matter can lead to interesting phenomenological consequences in core-collapse supernovae, e.g., triggering an explosion in spherically symmetric models. However, until now this explosion mechanism was only shown to be working for equations of state that are in contradiction with recent pulsar mass measurements. Here we identify that this explosion mechanism is related to the existence of a third family of compact stars that is present only in the hot, early stages of their evolution. Its existence is a result of unusual thermal properties of the two-phase coexistence region of the PT, e.g., characterized by a decrease of temperature with increasing density for isentropes, and which can be related to a negative slope of the PT line in the temperature-pressure phase diagram.

PACS numbers: 21.65.Qr, 25.75.Nq, 26.50.+x, 26.60.Kp

INTRODUCTION

The explosion mechanism of core-collapse supernovae (CCSNe) is a long-standing problem in astrophysics. The occurrence of quark matter in CCSNe can have interesting consequences in this respect: Sagert et al. [1] simulated a CCSN in spherical symmetry with a phase transition (PT) to quark matter that sets in at rather low densities. They found that at a certain point during the accretion in the post-bounce phase, when a critical fraction of quark matter is reached in the center, the nascent proto-compact star (PCS) loses its stability. A second collapse of the core is triggered. Once pure quark matter is reached in the center, the collapse halts and a second outgoing accretion shock is formed. It is strong enough to unbind the outer layers once it merges with the standing accretion shock, resulting in an explosion just due to the PT to quark matter. The passage of the second shock over the neutrino spheres leads to a second neutrino burst that could be measured with present-day neutrino detectors [2] and that would give an observational signature for the QCD PT in CCSNe.

However, the hybrid equations of state (EOSs) applied in [1] have maximum masses much below $2 M_{\odot}$, and are thus ruled out by the recent observations of pulsar masses around $2 M_{\odot}$ [3, 4]. In the subsequent works exploring this scenario [5–11], explosions could not be obtained if the maximum mass was sufficiently high. The required stiffening of the quark EOS leads to a weaker PT, and thus less pronounced features in the CCSN simulations. With the present investigation we are not yet able to give a definite answer to the question whether or not the QCD PT is still a viable CCSN explosion mechanism. However, here we give a new explanation for the occurrence of the

second collapse that can trigger an explosion. Furthermore we show that it is related to general features of the QCD PT.

A THIRD FAMILY OF PROTO-COMPACT STARS

Here, we consider two hybrid EOSs as representative examples. The B165 (B139) EOS uses a bag constant of $B^{1/4} = 165$ MeV (139 MeV), and has a maximum mass of $1.51 M_{\odot}$ ($2.08 M_{\odot}$). Only the former leads to explosions in spherical symmetry [1, 7, 11]. The high maximum mass of the B139 EOS is achieved by the inclusion of strong interactions with coupling constant $\alpha_S = 0.7$ [7, 12]. The hadronic parts of the hybrid EOSs are taken from [13, 14] (STOS98) for B139 and from [15] (STOS11) for B165. STOS98 and STOS11 are based on the same underlying model, but have numerical differences. Global charge neutrality was assumed for the PT and Gibbs conditions for phase equilibrium have been applied.

In Fig. 1 we show the mass-radius relations of the four EOSs, for various entropies per baryon S and in beta-equilibrium without neutrinos.¹ One observes an interesting feature for the hybrid EOSs: With increasing entropy, a second maximum develops, that eventually even becomes the global maximum. It is well known from cold compact stars (CSs) that stars on the branch between the

¹ We terminate the numerical integration of the Tolman-Oppenheimer-Volkoff equations at a pressure of 10^{30} erg/cm³, to avoid an unrealistically large contribution of a hot, low-density envelope.

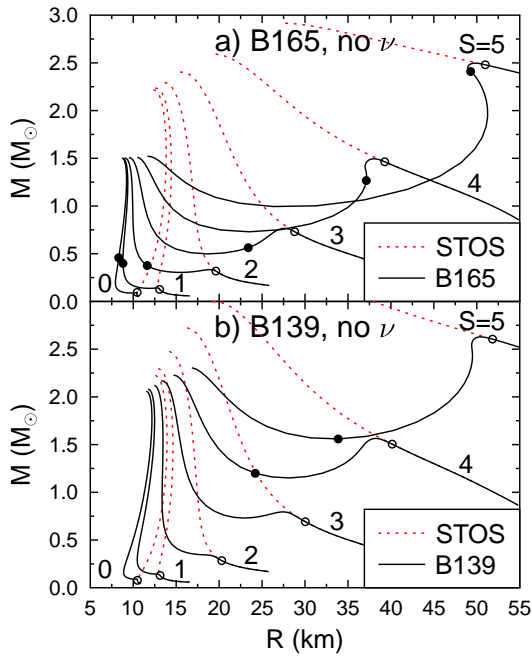


FIG. 1. (Color online) M - R -relations for different entropies per baryon S (indicated by the numbers in the figure) and in beta-equilibrium without neutrinos. a) B165 hybrid EOS (black solid lines) and STOS11 hadronic EOS (red dotted lines). b) B139 hybrid EOS (black solid lines) and STOS98 hadronic EOS (red dotted lines). Open (full) circles indicate that phase coexistence (pure quark matter) has been reached.

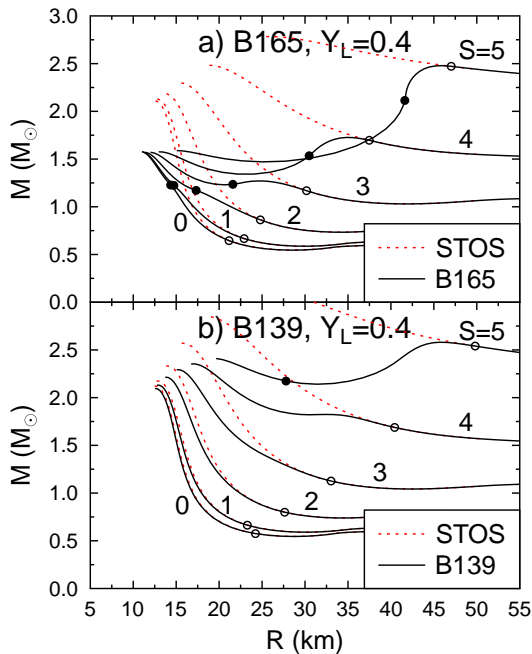


FIG. 2. As Fig. 1, but with completely trapped neutrinos in beta-equilibrium and a lepton fraction Y_L of 0.4.

right maximum and the local minimum are unstable with respect to radial perturbations [16]. In such a situation, where one has two different stable branches in addition to the one of white dwarfs, one speaks about a third family (TF) of CSs [16–18]. Solutions with different radii at equal masses are also called twins [16, 18]. In Fig. 1 we find the existence of the TF to depend on entropy; cf. [19]. It does not exist or is only very tiny for cold CSs and becomes more pronounced for increasing entropy. For B165 the TF branch appears for lower entropies and is generally more pronounced than for B139. It is also interesting that B139 hybrid stars do not contain pure quark matter, except at highest entropies. Note that for both EOSs for $S = 5$ there is a TF, but its maximum baryonic mass is below the one of the second family. In this case the TF cannot be reached by accretion, a collapse from the maximum of the second family would end in a black hole.

In PCSs one not only has finite temperature, respectively entropy, but also a finite fraction of neutrinos, which are typically completely trapped in the core. To identify their effect on the stability of PCS, in Fig. 2 we consider completely trapped neutrinos and a constant lepton fraction Y_L of 0.4. Neutrinos tend to decrease effects of the PT and the TF feature, because they give a similar contribution to the thermodynamic properties in both phases. In Fig. 2 a) the TF is only visible for $S \geq 3$ and in b) only for $S \geq 4$. However, it is not very realistic to assume a constant value of Y_L throughout the PCS. The results shown in Fig. 2 (Fig. 1) can be considered as an overestimate (underestimate) of the effect of neutrinos. A more realistic situation should be somewhere in between.

RELATION TO THE SECOND COLLAPSE

Fischer et al. [7] gave detailed explanations of the processes before and during the second collapse in the CCSN. Here, we complement them by connecting the second collapse with the existence of a TF of PCS as shown in Figs. 1 and 2. In CCSNe, in the absence of shocks and as long as neutrinos are trapped, S and Y_L are conserved quantities that are only advected with the matter, and their values are similar as in Figs. 1 and 2. During the ongoing accretion in the post-bounce phase, the mass and central density of the PCS increase continuously. One moves along an M - R -curve to the left. At a certain point in the evolution, the central density has increased so far that the mostly hadronic PCS reaches the maximum mass of the second family. At this point a second collapse is induced. It proceeds until high enough pressures on the TF branch are reached that counterbalance gravity. Afterwards, the second shock is formed that has the potential to explode the star, see [7].

Even though the kinetic energy of the second collapse is not directly converted into explosion energy, see [7],

it is interesting to look at the release of gravitational binding energy² for the TFs shown in the figures above. It ranges from 0 to $\sim 120 \times 10^{51}$ erg, where the values are increasing with S and decreasing with Y_L . For B139 they are generally lower than for B165, but for both EOSs they can exceed the typical explosion energy of a CCSN by orders of magnitude if entropies are high and the lepton fraction is low. The finding that the TF is much less pronounced for B139 is consistent with the result that this EOS did not lead to explosions in spherical CCSN simulations [11]. Only a pronounced TF of PCS seems to be favorable for explosions.

We remark that the collapse from the second to the third family of cold CSs has been studied already in the literature, see, e.g., [20]. In addition to the energy release, an accompanying neutrino and/or a gamma-ray burst is expected. Pagliara et al. [21] noted that also deleptonization can trigger a collapse from the second to the third family, which represents a related scenario. A collapse in rotating stars can also lead to the emission of gravitational waves [22].

UNUSUAL THERMAL PROPERTIES OF THE EOS INDUCED BY THE PT

For most EOS the pressure is increasing with temperature, consider, e.g., an ideal Maxwell-Boltzmann or Fermi-Dirac gas. However, it is also possible in special situations that $\partial P / \partial T|_{n_B} < 0$.³ In [23] this was called “abnormal thermodynamics” and it was pointed out in [23, 24] that such an unusual sign of a second cross derivative never occurs isolated, but is accompanied by a change of the sign of many other second cross derivatives. For example one has

$$\left. \frac{\partial P}{\partial T} \right|_{n_B} < 0 \Leftrightarrow \left. \frac{\partial T}{\partial n_B} \right|_S < 0. \quad (1)$$

In Fig. 3 we show the temperature as a function of baryon number density for several isentropes of the STOS11 and B165 EOSs. A region with negative slope $\partial T / \partial n_B|_S$ is present for all entropies. It shifts to lower densities and becomes more pronounced by increasing S . For the B139 EOS such a negative slope is only found for high values of S above 4. Generally, it occurs only inside the two-phase coexistence region of the PT, see Fig. 3. This unusual decrease of temperature due to the PT has also been found for various other hybrid EOSs, see, e.g., [7, 19, 25–27].

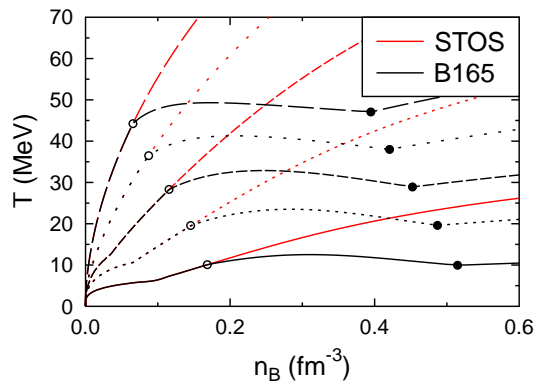


FIG. 3. Temperature as a function of baryon number density for isentropes with $S = 1, 2, 3, 4$, and 5 , (increasing from bottom to top) and $Y_L = 0.4$ for the B165 (black solid lines) and STOS11 (red dotted lines) EOSs. The open (full) circles mark the onset (end) of the PT.

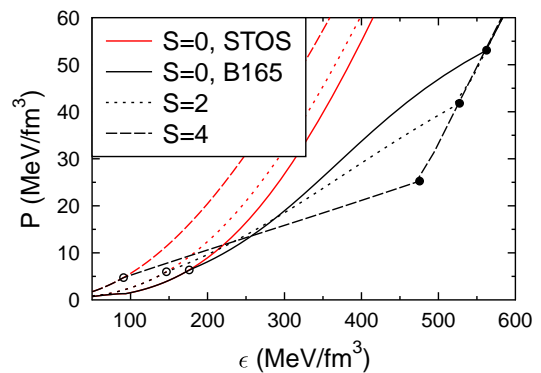


FIG. 4. (Color online) The pressure as a function of energy density for the B165 (black lines) and STOS11 (red lines) EOSs for $Y_L = 0.4$ and various entropies per baryon S . The open (full) circles mark the onset (end) of the PT.

This behavior has direct consequences on the stability of CSs. Let us consider the derivative

$$\left. \frac{\partial P}{\partial S} \right|_{\epsilon} = -T n_B \left(\frac{c_s}{c} \right)^2 + \frac{T}{C_V} \left. \frac{\partial P}{\partial T} \right|_{n_B}, \quad (2)$$

where ϵ is the energy density, c_s the speed of sound, c the speed of light, and C_V the heat capacity per baryon (see also [28]). If $\partial P / \partial S|_{\epsilon}$ is positive (negative), it corresponds to a stiffening (softening) of the EOS with increasing S . The first term, that is a relativistic correction, is always negative. Usually the second term is positive (as outlined above) and larger than the first one, and thus one has stiffening. However, whenever $\partial T / \partial n_B|_S < 0$ or equivalently $\partial P / \partial T|_{n_B} < 0$ and because $C_V > 0$, $\partial P / \partial S|_{\epsilon}$ will be negative and one has softening.

To illustrate this, In Fig. 4 we show the STOS (red curves) and B165 (black curves) EOSs for $Y_L = 0.4$ and entropies per baryon of $S = 0, 2$, and 4 . In the hadronic phase the pressure is always increasing with entropy. In the pure quark phase the pressure is only slightly in-

² It is given by the mass difference between the maximum of the second branch and a star with equal baryon number on the third branch.

³ Note that this does not violate thermodynamic stability.

creased by entropy. Outside the PT one thus has the normal behavior that the EOS is stiffened when it is heated. However, in a part of the PT around $\epsilon = 400 \text{ MeV/fm}^{-3}$ one has softening, i.e., the pressure is decreasing with entropy. The same effect was noted in [26] before.

The stiffening in the single phases and softening in the PT fits the behavior of the M - R relations discussed above: On the one hand, overall the maxima of the second and, if present, also of the third family, are increasing with entropy. On the other hand, stars whose central part has just entered the PT get unstable if entropies are sufficiently high. One can conclude that the unusual thermal properties of the PT, characterized, e.g., by $\partial T/\partial n_B|_S < 0$, are responsible for the observed TF features.

RELATION TO THE QCD PHASE DIAGRAM

Let us consider symmetric nuclear matter without strangeness. In this case, the pressure in the coexistence region of the PT is solely a function of temperature⁴ and thus we can identify $\partial P/\partial T|_{n_B} = dP/dT|_{\text{PT}}$, where the latter quantity denotes the slope of the PT line in the temperature-pressure phase diagram. By using Eq. (2) one can thus relate the QCD phase diagram with a possible softening or stiffening of the EOS with increasing entropy.

In Refs. [23, 24, 29, 31] it was shown that the slope $dP/dT|_{\text{PT}}$ is negative for the QCD PT, and positive for the liquid-gas phase transition (LGPT) of nuclear matter (see also [26, 27, 32]). This qualitative difference can be used to introduce a subclassification of first-order PTs: in [23, 24] they were called *entropic* and *enthalpic*, respectively. For possible experimental signatures of this property in heavy-ion collisions see [31]. Also for the hybrid EOSs employed in the present study, we have found a negative slope, i.e., that the QCD PT is entropic. For such a PT we always expect the unusual thermal properties outlined above (e.g., $\partial T/\partial n_B|_S < 0$) and thus a softening of the EOS with increasing entropy $\partial P/\partial S|_\epsilon < 0$. We remark that the general relation between the slope of the PT line and the unusual behavior of the second cross derivatives was first noted in [24].

The reason for this special property of the QCD PT can be identified by using the Clausius-Clapeyron equation [19, 23, 24, 29, 32]:

$$\left. \frac{dP}{dT} \right|_{\text{PT}} = \frac{S^I - S^{II}}{1/n_B^I - 1/n_B^{II}}, \quad (3)$$

where I and II denote the two phases in coexistence, whereas $n_B^I < n_B^{II}$. One has $S^I < S^{II} \leftrightarrow dP/dT|_{\text{PT}} < 0$.

The QCD PT has a negative slope (i.e., $dP/dT|_{\text{PT}} < 0$) because the quark phase has a higher entropy per baryon than the hadronic phase, which can also be inferred from Fig. 3. The basic degrees of freedom in the quark phase (i.e., the chirally restored quarks) have much lower masses than the hadrons, and thus are more relativistic. This increases the specific heat and the entropy per baryon; cf. [25]. In the LGPT the opposite is the case: the denser phase (the liquid) has the lower entropy per baryon, $S^{II} < S^I$. This PT does not change the structure of the constituent particles; only the density, entropy, energy, and possibly the asymmetry of the two phases in coexistence are different.

We remark that the hybrid EOSs considered in the present study contain strange quarks in weak equilibrium, a leptonic component to maintain charge neutrality, and generally we consider asymmetric systems. As a consequence, it is not possible to relate $\partial P/\partial S|_\epsilon$ directly with the slope of a PT line and only parts of the coexistence region of the B165 and B139 EOSs show anomalous thermodynamics.

SUMMARY AND CONCLUSIONS

In the present study, we have investigated effects of the QCD PT in CCSNe. We found that the explosions reported in [1, 7] can be explained as a transition from a second to a third family of PCS. If we interpret the first collapse of the iron core of the progenitor star as a transition from the first to the second family of CSs, massive progenitor stars undergo at least one transition between different families of CSs. If also the second transition from the second to the third family takes place this can lead to a CCSN explosion.

Interestingly, the TF feature was only very tiny in the case of cold CSs, and found to be enhanced with increasing entropy. This was explained as a result of unusual thermal properties of the EOS induced by the PT. It is characterized, e.g., by a decrease of temperature with density along isentropes in the PT, $\partial T/\partial n_B|_S < 0$, and directly implies a softening of the EOS for increasing entropy, $\partial P/\partial S|_\epsilon < 0$. In this situation one can generally expect that a pronounced TF of PCS will emerge for high enough entropies. One could say that unusual thermal properties of the PT also favor unusual behavior in the M - R -relation.

We showed that the unusual thermal properties are related to a negative slope of the PT line in the temperature-pressure plane, which in turn can be related to higher entropies per baryon in the quark than in the hadronic phase. From our perspective it is quite remarkable that the M - R -relation and CCSN explosions can be linked with the phase diagram of QCD in this way, whose structure is one of the key issues in the physics of strongly interacting matter. It would be very interesting

⁴ In [29], this was denoted as a “congruent PT”, using the terminology of [30].

if the slope of the PT line could be constrained by heavy-ion collisions or lattice QCD calculations in the future, cf. [31].

However, we remark that the special thermal properties can also be present without a first-order PT: E.g., in [33] a negative value of $\partial T/\partial n_B|_S$ was identified for a cross-over transition from hadronic to quark matter and in [34] for a hadronic EOS including hyperons and deltas.

We close with a few comments regarding the question if one can have a sufficiently strong TF feature in hot PCSs to trigger explosions as in [1] and a sufficiently high maximum mass of cold CSs at the same time. It seems to be that one needs pure quark matter in the core of the most massive CSs and a PT that does not occur at too high densities. Of course it is favorable if there is already a TF for cold CSs, cf. [35, 36]. An extended phase-coexistence region, as seen for the B139 EOS, tends to reduce effects of the PT. This could be modified by the assumption of local charge neutrality. Besides that, one should explore different models for hadronic and quark matter, whereas their thermal properties are of particular relevance.

Even if explosions cannot be obtained in spherical symmetry, there could be interesting effects of the QCD PT in multi-dimensional hydrodynamic simulations of CCSNe (cf. Refs. [37, 38] and references therein), which has not been investigated so far. For example, Yudin et al. [28] recently reported that the unusual thermal properties of the QCD PT can induce a special convective instability.

The authors would like to thank J. Schaffner-Bielich, G. Pagliara, I. Sagert, and T. Fischer for their useful comments and discussions, and I. Sagert for providing the hybrid EOS tables used in this work. The research leading to this paper has received funding from the Swiss National Science Foundation, and the European Research Council under the European Union's Seventh Framework Programme (FP7/2007-2013) / ERC Advanced Grant Agreement N 321263 - FISH. Partial support comes from "NewCompStar", COST Action MP1304. I.I. acknowledges support of Russian Scientific Fund, Grant No: 14-50-00124.

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